Journal of Engineering Mathematics **31:** 397–409, 1997.

Dynamics of a pulsating spray-diffusion flame

J.B. GREENBERG and R. COHEN
Faculty of Aerospace Engineering, Technion – Israel Institute of Technology, Haifa 32000, Israel

Received 11 January 1995; accepted in revised form 21 May 1996

Key words: Burke-Schumann, spray-diffusion, flames, Damkohler number

Abstract. A mathematical model of a fluctuating Burke-Schumann spray-diffusion flame is presented. A sectional approach is utilized to describe the spray of droplets. An analytic solution is presented for a periodically alternating supply of liquid/vapor fuel. Calculated data highlight the dynamics of the flame envelope under the fluctuating supply condition, as well as its sensitivity to liquid-fuel volatility and droplet size (expressed through a nondimensional Damkohler number for evaporation). The predicted flame shape dynamics is rather similar, qualitatively, to some experimental evidence in which flame growth and collapse are observed.

1. Introduction

In combustion systems utilizing liquid fuel the liquid is generally atomized to produce a cloud of droplets that subsequently vaporize and burn either individually or, more frequently, as a cluster. Desirable combustion characteristics such as combustion efficiency, minimal pollutant production, avoidance of unwanted hot-spots etc. are critically determined by the nature of the behavior of the cloud of drops. It is a rather well established fact that significant impairing of combustion efficiency can occur owing to distortion of fuel-spray patterns brought on by even slight nozzle-lip imperfections. Characterization of the influence of such spatial nonuniformities has indeed begun to receive some experimental and theoretical attention (Ortman and Lefebvre, [1], Rosjford and Russell [2], Rosjford and Eckerle, [3], Greenberg [4]). It is probably a truism to add that distortion of fuel patternation is probably not due to a steady-state process, but rather the result of a time-dependent mechanism involved in the inherent flow instability that is at work during atomization of the liquid fuel. In fact, it is possibly this mechanism, coupled to dominant turbulent effects (Chigier et al., [5], Bellan and Harstad [6]), that produces discrete clusters or groups of droplets. Thus, temporally varying concentrations of droplets (and, hence, of vapors) can be envisaged as profoundly affecting flame characteristics under certain circumstances in as critical a fashion as spatial nonuniform patterns do. Nevertheless, much experimental research still relies on assumptions of on-the-average axisymmetry or time-averaging over scales that do not capture the details of the aforementioned fluctuations (e.g. Presser *et al*. [7], Mao *et al*. [8]). However, this does not detract from the fact that much relevant and useful information on spray combustion can be gleaned from such steady-state studies.

Some qualitative evidence of the way in which a spray-diffusion flame may fluctuate in time can be found in a sequence of photographs presented by Dombrowski *et al*. [9]). A schematic sketch of the flame shapes at different instants is shown in Figure 1. The spray of droplets is injected from below. The details of the set-up can be found in the original paper. What is of interest here are the characteristic features of the sequence of flame shapes, in particular above the lower "stem" of the flame. In the first figure on the left the apparent shape

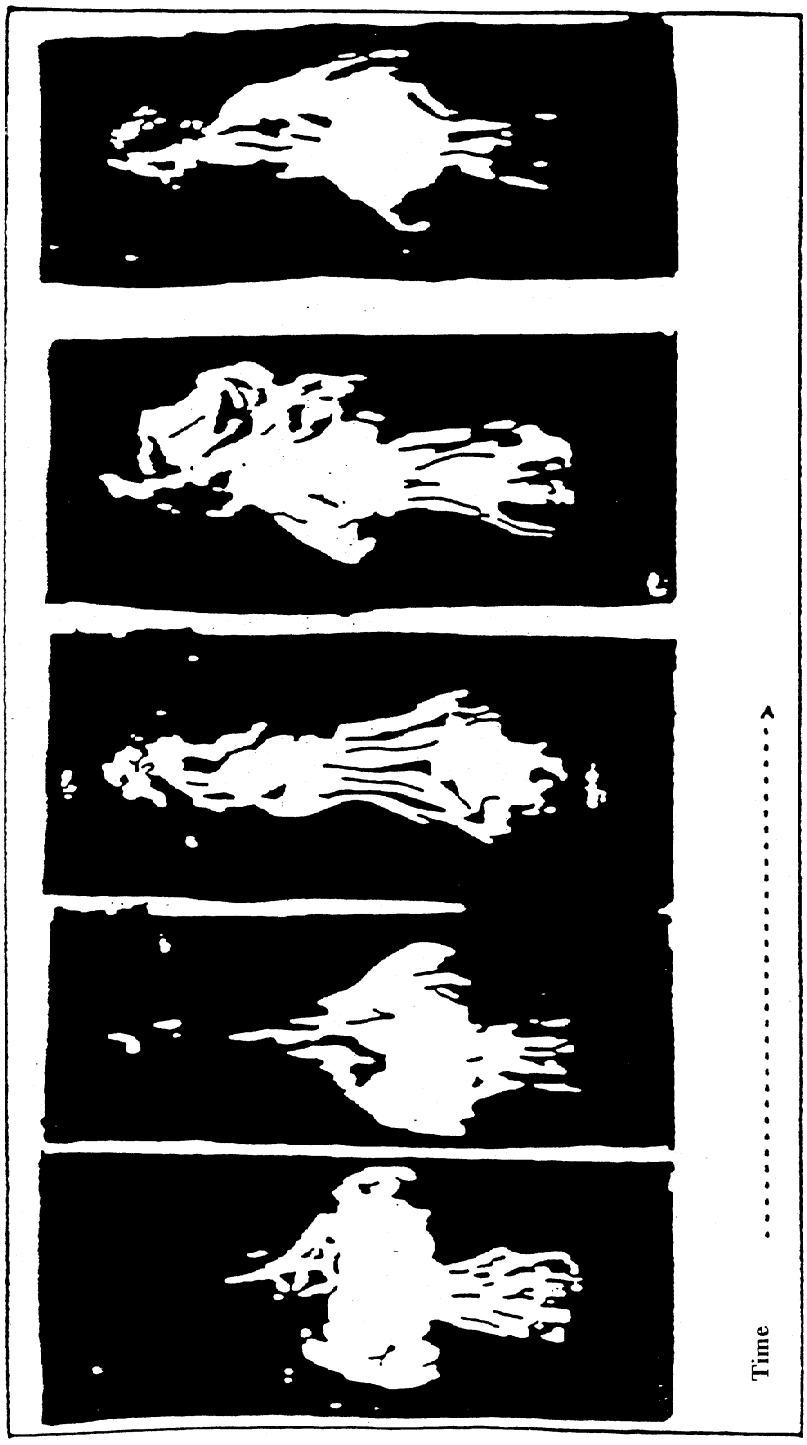

Figure 1. A Sketch of Flame Fluctuations (based on Figure 1 of Dombrowski *et al.* (1974)); time between frames is reported to be 30ms.

is somewhat bulbous. When moving to the right, we observe a slight thinning out of the flame which progresses further, as seen in the next frame. Subsequently, a sort of mushroom shape begins to reappear, followed by what is probably the beginning of the reconstruction of the original shape, seen in the first frame. There seems to be a mechanism producing a pulsating type of behavior of the flame shape, although it is not clear exactly what this mechanism is.

Levy and Bulzan [10] have conducted an experimental investigation of a laminar Burke-Schumann spray-diffusion flame. Interestingly, they have found that under certain operating conditions a steady-state flame cannot be produced – rather, the spray flame grows in size, then appears to collapse (although not to complete extinction), "inflates" again, and so on in a regular periodic fashion.

In the context of gas flames Kimura [11] has discussed intense flame oscillations of the order of 15 cps. These oscillations are related to the phenomenon known as flame flicker [12]. Kimura's analysis leads him to the conclusion that the flame oscillations are induced by the instability of the laminar-jet flows. Buckmaster and Peters [12] have shown that the almost universal frequency of these gas diffusion-flame oscillations is a direct result of buoyancy effects. However, for the spray-diffusion flames the oscillations found by Levy and Bulzan [10] are of quite a different nature, having a frequency of between 1–5 Hz. Levy and Bulzan suggest that buoyancy effects are responsible for their spray-flame pulsations. However, their hypothesis, although feasible, must remain speculative until more extensive parametrically controlled experimental data become available. Indeed, recent computations of this sort of flame, including buoyancy effects, have failed to simulate the observed oscillations [13]. Nevertheless, this fairly sparse, yet clear evidence of a fluctuating spray flame envelope provides further motivation for our current work.

In the present paper we introduce a model of a spray-diffusion flame and its evolution under conditions of temporally varying fuel droplet and vapor supply, without specifying a particular mechanism producing these variations. Our aim is to examine the dynamics of the flame envelope under these conditions, as well as to look into its sensitivity to liquid-fuel volatility and droplet size. We solve the mathematical model analytically and, in the final section, calculated results are presented, showing the influence of the aforementioned parameters on the temporal fluctuations of spray-diffusion flames.

2. Assumptions of the model

The diffusion flame under consideration is formed as the result of diffusive intermixing of two gaseous streams exiting from an inner and outer duct (see Figure 2). The outer duct contains air (supplied at a constant rate), whilst a temporally varying mixture of inert gas, gas vapor and fuel droplets flow from the inner one. Following Burke and Schumann [14], we assume that the average velocity of the two streams is approximately equal and constant, even though the composition of the inner one changes with time. The fuel droplets are presumed to be injected into the inner duct at a sufficiently large distance upstream, so that at its exit they have already undergone initial breakup and partial vaporization and have settled into a motion at a velocity equal to that of the gas stream in which they are suspended. This assumption is often adopted in analyzing spray characteristics and finds supportive evidence in experimental data (Yule *et al*. [15]) and theoretical and experimental comparisons (Tambour [16]). The diffusion coefficients of all gaseous species are taken as equal and constant and the Lewis number is assumed to be unity. The detailed flame structure is not considered here; rather the Burke-Schumann criterion is adopted whereby the flame front is defined as the

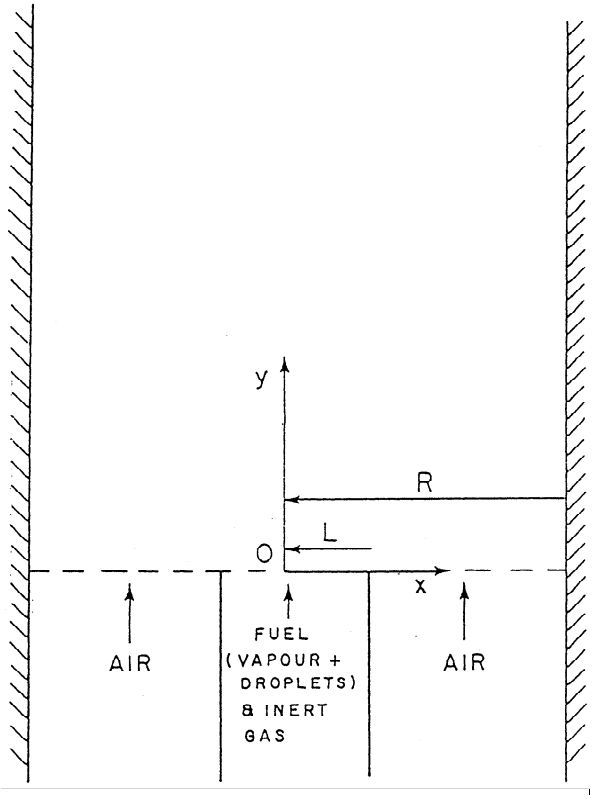

Figure 2. Configuration for Spray Diffusion Flame Formation.

locus of points at which the gaseous fuel and oxidant are in stoichiometric ratio and where instantaneous chemical reaction occurs. (This effectively means that the analysis given is an $O(1)$ outer solution in the limit of infinite chemical Damkohler number.)

3. Spray equations

The sectional approach of modelling polydisperse sprays is herein adopted (Tambour [16]). As details of its development have been copiously recorded elsewhere, (Tambour [16], Greenberg et al. [17]) they will not be repeated here. It suffices to say that the approach is based on dividing the pointwise droplet-number distribution function into distinct size sections from which so-called averaged sectional conservation equations are rigorously derived. In addition, in order to simplify the analysis, the concept of a quasi-monodisperse spray (Greenberg *et al*. [18]) will be utilized here, although a multisectional polydisperse spray model can be accommodated at the expense of algebraic complexity.

Under the assumptions of the previous section the single sectional spray conservation equation is given by:

$$\frac{\partial m_d}{\partial t} + u\frac{\partial m_d}{\partial y} = -\bar{C}m_d, \quad 0 \leq x \leq L, \tag{1}$$

where m_d is the mass fraction of liquid fuel (droplets). The sectional vaporization coefficient, $\bar{C}$, turns out to be of the form (Tambour, [16]):

$$\bar{C} = \left(\frac{3\pi^2}{4}\right)^{1/3} E(T) \left[\frac{3}{4}(v_H^{4/3} - v_L^{4/3}) + v_L^{4/3}\right] \times (v_H^2 - v_L^2)^{-1/2}, \tag{2}$$

in which v_H, v_L are the volume of the largest and smallest droplets in the spray under consideration and $E(T)$ is a (possibly) temperature-dependent coefficient of evaporation. The latter may be supplied based on theoretical or experimental data (Polymeropoulos [19]). Underlying the expression for $\bar{C}$ is the use of the d^2-law for evaporation. Reasonably accurate estimates of droplet size and vaporization time do provide evidence of the validity of this law, even under transient temperature conditions (Law and Sirignano [20], Law [21], Labowsky [22]). The location of the spray in the current model is in the far-field region (Chigier [23, pp. 248–256], Greenberg and Tambour [24], Tambour et al. [25]). Here, the transient stage in the droplets' evaporation lifetime has basically ceased, thus providing further justification for the d^2-law. If, in addition, we limit the liquid fuel to one that is fairly volatile (e.g. n-heptane, iso-octane), then Equation (2) is well vindicated.

4. Gas phase equations

Under the aforementioned assumptions the conservation equations for the mass fractions of the gaseous phase are:

$$\frac{\partial m_i}{\partial t} + u\frac{\partial m_i}{\partial y} = D\frac{\partial^2 m_i}{\partial x^2} + S_{\text{i, react}} + S_{\text{i, vap}}\mathcal{H}(L - x), \tag{3}$$

where there is one equation of this form for each chemical species present and $\mathcal{H}$ is the Heaviside function which we introduce to indicate that no droplets stray in the transverse direction beyond $x = L$. The source terms stem from chemical reaction and vaporization, respectively.

We now define the Schwab-Zeldovich variable:

$$m = m_{\text{g,fuel}} - m_{\text{O}_2}/\nu \tag{4}$$

and, after suitable combination of Equations (3) for the fuel vapor and oxygen, we obtain:

$$\frac{\partial m}{\partial t} + u\frac{\partial m}{\partial y} = D\frac{\partial^2 m}{\partial x^2} + S_{\text{fuel,vap}}. \tag{5}$$

The energy equation is given by

$$C_p\frac{\partial T}{\partial t} + C_p u\frac{\partial T}{\partial y} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + QS_F. \tag{6}$$

5. Boundary conditions

For the problem at hand we wish to simulate periodically supplied liquid and vaporized fuel. The oxygen supply will be held constant. Thus, the boundary conditions we adopt have the form:

(i) For the spray of liquid fuel:

$$y = 0; t \geq 0 \quad m_d(t) = \begin{cases} m_d(t), & 0 \leq x \leq L \\ 0 & L \leq x \leq R \end{cases};$$

(ii) For the gas phase:

$$y = 0; \quad t \geq 0 \begin{cases} m = 1 - m_d(t), & 0 \leq x \leq L \\ -m_{\mathrm{O_2o}}/\nu & L \leq x \leq R \end{cases}$$

$$y = 0; \quad t \geq 0 \quad T = T_{\mathrm{o}}, \quad 0 \leq x \leq R.$$

At the outer wall of the duct and along the axis of symmetry:

$$x = 0, R; \; t \geq 0 : \; \frac{\partial m}{\partial x} = \frac{\partial T}{\partial x} = 0.$$

Finally, the conditions at $t = 0$ are specified:

$$y \geq 0; t = 0 \quad m = m_d = 0, \quad T = T_{\mathrm{o}}.$$

The exact functional form of the supply of fuel, $m_d(t)$, will be described after the next section.

6. Normalization of governing equations

We now reformulate our problem in terms of normalized quantities, so as to facilitate a parametric analysis. Define the following quantities:

$$(\xi, \eta, c) = \left(\frac{x}{R}, \frac{yD}{uR^2}, \frac{L}{R} \right), \tag{7}$$

$$\tau = tD/R^2, \tag{8}$$

$$(\gamma, \gamma_d, V) = (m, m_d, m_{\mathrm{O_2o}}/\nu)/m_{\mathrm{totfuel}}, \tag{9}$$

$$T^* = (T - T_{\mathrm{o}})C_p/Q, \tag{10}$$

where m_{totfuel} is the total mass fraction of fuel (*i.e.* liquid droplets + vapors) at the exit of the inner duct. Use of these definitions in (1), (5) and (6) produces the governing two-phase equations:

$$\frac{\partial \gamma_d}{\partial \tau} + \frac{\partial \gamma_d}{\partial \eta} = -\Delta\gamma_d, \quad 0 \leq \xi \leq c,$$

$$\frac{\partial \gamma}{\partial \tau} + \frac{\partial \gamma}{\partial \eta} = \frac{\partial^2 \gamma}{\partial \xi^2} + \Delta\gamma_d \mathcal{H}(c - \xi),$$

$$\frac{\partial T^*}{\partial \tau} + \frac{\partial T^*}{\partial \eta} = \frac{\partial^2 T^*}{\partial \xi^2} + \frac{R^2 S_F}{D}. \tag{11}$$

Combining (11) with (3) (written in normalized form) for the fuel and defining

$$\gamma_T = m_{\mathrm{g,fuel}} + T^*, \tag{12}$$

we are able to formulate the following equation for γ_T:

$$\frac{\partial \gamma_T}{\partial \tau} + \frac{\partial \gamma_T}{\partial \eta} = \frac{\partial^2 \gamma_T}{\partial \xi^2} + \Delta \gamma_d \mathcal{K}(c - \xi). \tag{13}$$

The parameter Δ is defined as

$$\Delta = \bar{C} R^2 / D. \tag{14}$$

As we previously noted (Greenberg [26,27]), this represents the ratio of a diffusion-to-vaporization time and, as such, will be referred to as a Damkohler number for vaporization. For later reference it is worth noting the significance of this number. From the form of $\bar{C}$ (2) we can see that, for a fixed value of the diffusion coefficient and a given geometric configuration, a large value of Δ will reflect either a very volatile liquid fuel or small droplets in the spray. The converse is true for small values of Δ – such values will result from large droplets and/or a less volatile fuel. In this way the single parameter Δ enables one to capture a variety of situations of interest.

In a similar manner we recast the boundary conditions into nondimensional form:

$$\begin{aligned}
&\tau = 0,\ \eta \geq 0 \quad \gamma_d = \gamma = \gamma_T = 0 && 0 \leq \xi \leq 1,\\
&\tau > 0;\ \eta = 0 \quad \begin{cases} \gamma_d = f(\tau) & 0 \leq \xi \leq 1 \\ 0 & c \leq \xi \leq 1 \end{cases},\\
&\tau > 0;\ \eta = 0 \quad \begin{cases} \gamma = 1 - f(\tau) & 0 \leq \xi \leq c \\ -V & c \leq \xi \leq 1 \end{cases},\\
&\tau > 0;\ \eta = 0 \quad \begin{cases} \gamma_T = 1 - f(\tau) & 0 \leq \xi \leq c \\ 0 & c \leq \xi \leq 1 \end{cases},\\
&\eta > 0 \quad \tau \geq 0;\ \xi = 0, 1: \quad \frac{\partial \gamma}{\partial \xi} = \frac{\partial \gamma_T}{\partial \xi} = 0.
\end{aligned}$$

7. Fluctuating supply of liquid and vapors of fuel

In order to simulate a fluctuating supply of liquid and vaporized fuel, a specific form must be given for $f(\tau)$. In view of the preliminary results of Levy and Bulzan [10] mentioned before we arbitrarily choose the following functional form:

$$f(\tau) = \frac{\bar{f}}{2}(1 - \cos \Omega \tau), \tag{15}$$

where $\bar{f}$ is a constant and Ω is the nondimensional frequency.

We recall that, at the present juncture, our aim is to try to model the flame envelope fluctuations with as simple a model as possible, without addressing the question of the physical mechanism producing them. For this purpose (15) has turned out to be satisfactory.

8. Solution for Burke-Schumann pulsating spray flame

The solution of the governing equations subject to the given boundary conditions is readily achieved by means of, for example, integral transform methods.

Solving first for the droplets, we obtain:

$$\gamma_d = \begin{cases} \mathrm{e}^{-\Delta\eta} f(\tau-\eta)\mathcal{H}(\tau-\eta), & 0 \le \xi \le c \\ 0, & c \le \xi \le 1 \end{cases}.$$

Armed with this solution for γ_d, we substitute this in (10) and (13) and solve to obtain:

$$\gamma(\tau,\eta,\xi) = \left\{ c - V(1-c) - c\,\mathrm{e}^{-\Delta\eta} f(\tau-\eta) + \sum_{n=1}^{\infty} \left[2(1 - f(\tau-\eta) + V)\,\mathrm{e}^{-(n\pi)^2\eta} + \frac{2\Delta}{(n\pi)^2 - \Delta} f(\tau-\eta) \left[\mathrm{e}^{-\Delta\eta} - \mathrm{e}^{-(n\pi)^2\eta} \right] \right] \frac{\sin n\pi c}{n\pi} \cos n\pi\xi \right\} \mathcal{H}(\tau-\eta). \tag{16}$$

γ_T is simply the same expression as γ with V set equal to zero.

Note that it is possible to determine the mass fractions of the fuel vapor and oxygen and the temperature in the entire solution domain from γ and γ_T. The diffusion flame, in the Burke-Schumann infinite chemical Damkohler limit, separates an oxygen-free region, $\mathcal{C}_F$, from a fuel-vapor-free region, $\mathcal{C}_{\mathrm{o}}$. Hence, in $\mathcal{C}_F, \gamma$ is simply the mass fraction of fuel vapor, whilst in $\mathcal{C}_o$ it enables the oxygen-mass fraction to be computed (see (4)). Similarly, knowing γ_F and using (12), we may extract the temperature. We determine the flame location at each instant in time by computing those points (ξ,η) for which $\gamma = 0$, as in Burke-Schumann's classical model. Before turning to our computed results, we believe it to be worth noting that the aforementioned solution reduces to the Burke-Schumann solution if $f = 0$ or if $\Delta \to \infty$, when a steady-state solution is obtained. This solution is also an extension of the steady-state Burke-Schumann spray flame of Greenberg [26].

9. Results and Discussion

The chemical kinetic scheme employed concerns the burning of n-heptane, C_7H_{16}, according to the global reaction

$$C_7H_{16} + 11O_2 \to 7CO_2 + 8H_2O.$$

For all results to be discussed the value of V was taken to be 0.306. The parameter $\bar{f}$ was assigned the value of unity, so that $f(\tau)$ would vary between 0 and 1 (i.e. a temporal variation between no liquid fuel and only liquid fuel). Further, c was taken to be 1/6.

In Figure 3 we present a series of results showing the evolution of a spray-diffusion flame front (only half of the flame is shown due to axial symmetry). A low value of $\Delta = 1$ was taken with the frequency factor $\Omega = 3\pi$. We may understand the collapse of the flame and its resurrection by referring to the behavior of $f(\tau)$ (15) i.e., the way in which the periodically alternating fuel vapor/liquid fuel arrives at the exit of the inner tube ($\eta = 0$). For this low value of the vaporization Damkohler number the fuel droplets vaporize slowly, so that, as long as there is an appreciable mass fraction of them present, insufficient fuel vapor is available to support a developed flame. When $\tau = \frac{1}{3}$, then $f(\frac{1}{3}) = 1$, so that $\gamma = 1 - f(\tau) = 0$. Thus, there is effectively hardly any combustion (see the frames for $\tau = 0.3, 0.4$). On the other hand for $\tau = \frac{2}{3}$ we have $f(\frac{2}{3}) = 0$, so that only fuel vapor is emerging from the inner duct at this point in time. The flame shape may then be expected to be most similar to that of the classical steady-state Burke-Schumann gaseous flame. Indeed, for $\tau = 0.66$ the last flame shape in Figure 3 is as we anticipated. The cycle repeats itself with the gradual collapse of the "fully

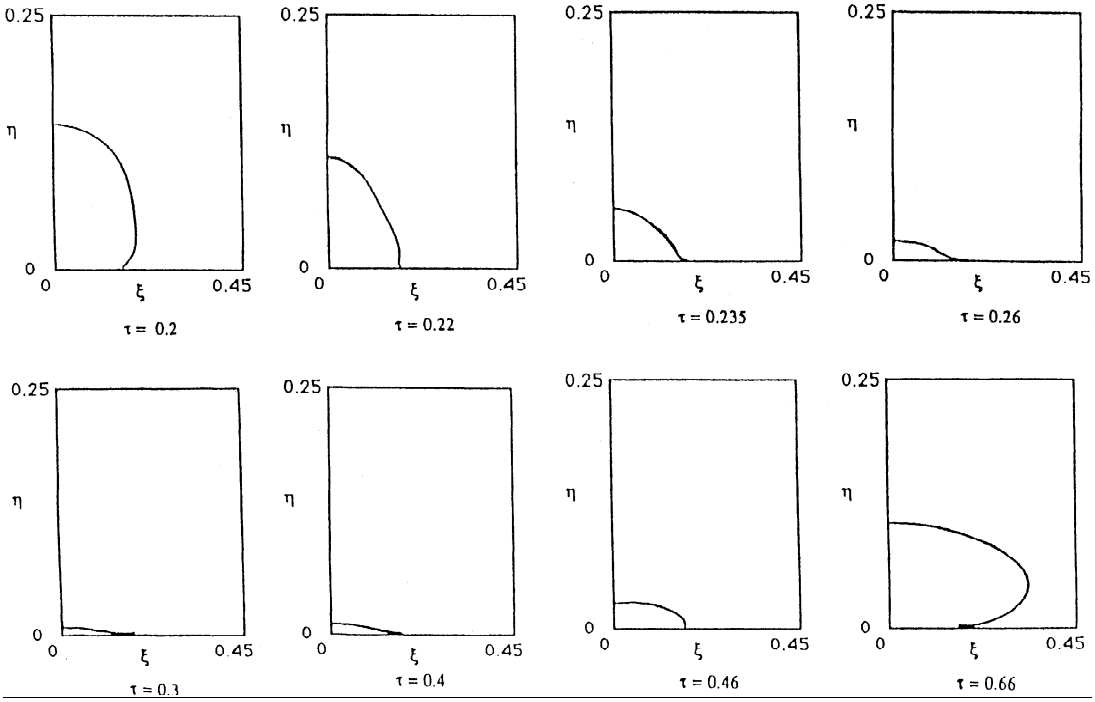

Figure 3. Stages in the Evolution of a Spray Diffusion Flame Front; Data: vaporization Damkohler number, $\Delta = 1$; Frequency of Fuel Supply Fluctuations, $\Omega = 3\pi$; Other Conditions as in Text.

developed" flame. This collapse is characterized by a thinning out of the width of the flame, accompanied by a reduction in its height brought on by a general decrease in the availability of fuel vapor with an increase in the presence of liquid fuel that is not vaporizing rapidly enough to maintain the combustion (see the frames relating to times $0.2 \leq \tau \leq 0.3$). Although we do not suggest that our simplified model provides an explanation of the probably nonlinear mechanism whereby the flame oscillations occur, we point out that there is quite a striking qualitative resemblance between this calculated behavior and the results of Dombrowski et al. [9] (see Figure 1).

We now consider the sensitivity of the afore-described behavior to the value of Δ. The results in Figure 4 were calculated for a Damkohler number of 10. Thus, the fuel may be more volatile and/or the droplets in the spray smaller than in our previous case. First of all, a general comment is appropriate. Comparison between Figure 3 and Figure 4 indicates that in the latter case the heights of the fully (or near-fully) developed flames is greater than in the former. A detailed comparison at different points in time reveals that the ratio of heights can be as much as 20 (at $\tau = 0.3$). The larger vaporization Damkohler number here ensures that more fuel vapor is supplied to the flame front, so that its collapse is not as pronounced as when $\Delta = 1$. In fact, following the time-sequence of solutions, we see that, as the maximum instantaneous value of $f(\tau)$ is approached, the flame narrows out at its stem, becoming concave there, whereas downstream it retains its convex shape. This is due to a sort of choking effect – droplets issuing from the inner tube need to travel a finite distance downstream to vaporize. As the flux of droplets increases, less vapor is available in the immediate vicinity of the exit of the inner tube, so that a local collapse of the flame occurs. Further downstream vapor is already available, so that the flame retains its original convex shape. As the large flux of droplets

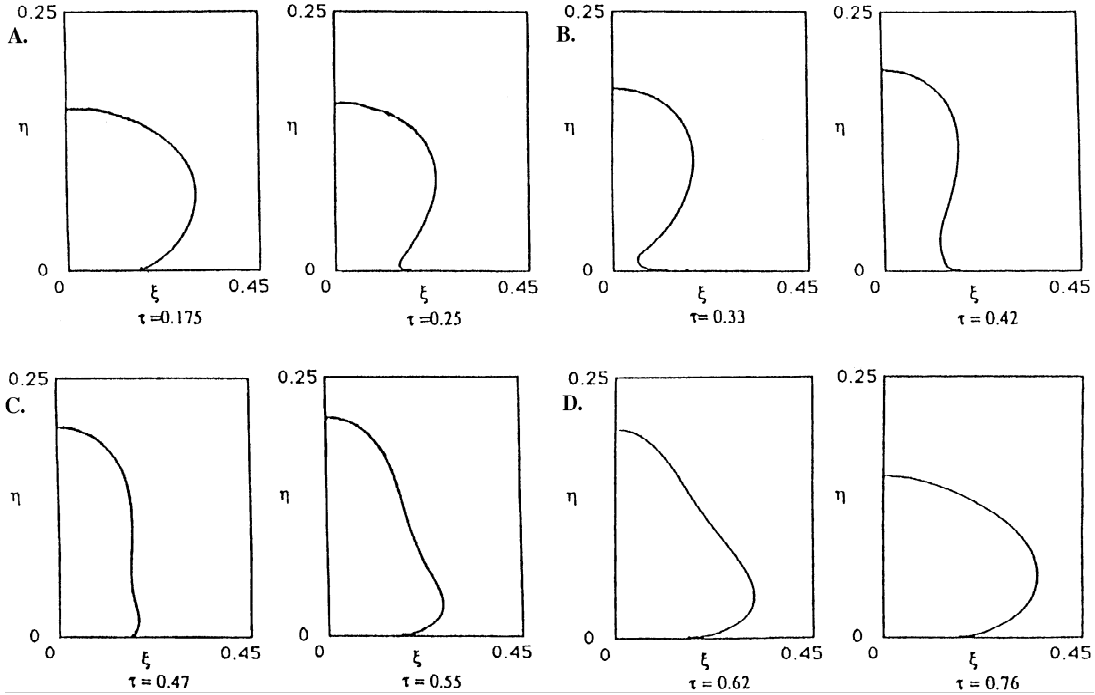

Figure 4. Stages in the Evolution of a Spray Diffusion Flame Front; Data: vaporization Damkohler number, $\Delta = 10$; Frequency of Fuel Supply Fluctuations, $\Omega = 3\pi$; Other Conditions as in Text.

travels downstream, they vaporize as well and the convexity is reduced since the droplets are only depositing their vapor in the region $\eta > 0, 0 \leq \xi \leq c$ (see time frames $\tau = 0.42, 0.47$). The "side" of the flame becomes almost vertical, and the flame height increases. Subsequently, the flux of gaseous fuel exiting from the inner tube reaches a maximum and the flame tends to return to its convex shape, with a decrease in height (time frames $\tau = 0.55, 0.62, 0.76$). This case clearly exemplifies the way in which the flame envelope can vary depending on whether fuel vapor is immediately available for combustion or whether it becomes available downstream as the cloud of droplets gradually vaporizes.

Further insight into the role of the spray in these pulsating flames is afforded by the sequence of time frames in Figure 5 in which the mass fraction of liquid fuel, γ_d, is plotted as a function of η at instants in time corresponding to those in Figure 4. At $\tau = 0.175$ the vast majority of droplets have evaporated before reaching the flame front, the height of the flame being approximately $\eta = 0.16$. In the next frame, $\tau = 0.25$, the initial mass fraction of liquid fuel is almost 0.9, and it is clear that droplets survive the flame front (at $\eta = 0.163$). Implicit in our model, (10), is the possibility of post-homogeneous flame-front burning of individual droplets in diffusive envelope flames surrounding them. The rate of this heterogeneous droplet combustion is controlled by the rate of vaporization. Evidence of this sort of heterogeneous combustion can be found in the experimental observations of Chen and Gomez [28]. As time progresses, more droplets survive the flame front (see time frames $\tau = 0.33 - 0.55$), even though the supply is diminishing at the exit of the inner tube. It has been demonstrated (Greenberg and Shpilberg [29]) that the post-flame individual droplet burning depletes the oxygen available for combustion at the homogeneous flame front, thus leading to taller flames. This is indeed the trend observed in the time period $0.33 \leq \tau \leq 0.55$ (see Figure 4). In the

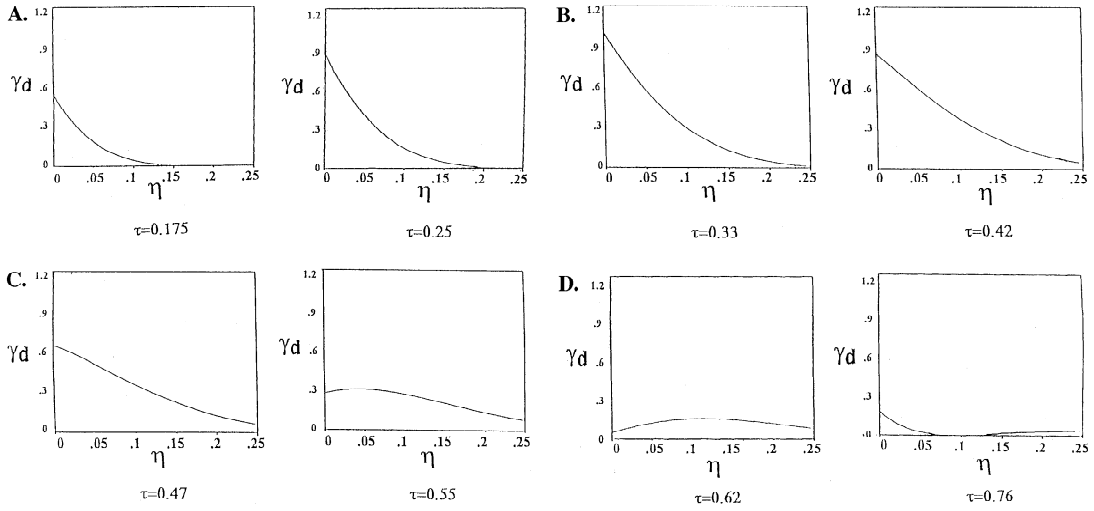

Figure 5. Spray Behaviour During Evolution of a Spray Diffusion Flame; Data: vaporization Damkohler number, $\Delta = 10$; Frequency of Fuel Supply Fluctuations, $\Omega = 3\pi$; Other Conditions as in Text.

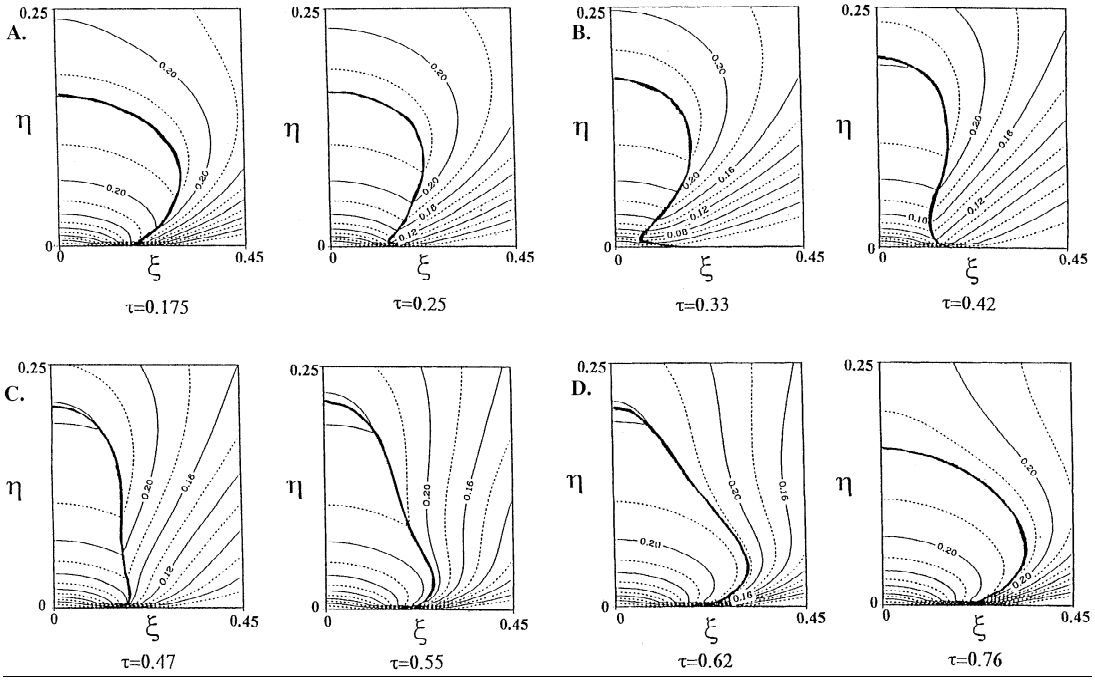

Figure 6. Normalized Temperature Contours During Evolution of a Spray Diffusion Flame; Data: vaporization Damkohler number, $\Delta = 10$; Frequency of Fuel Supply Fluctuations, $\Omega = 3\pi$; Other Conditions as in Text. The thick line marks the flame shape.

frames $\tau = 0.62, 0.70$ we observe the presence of low concentrations of droplets and, hence, high concentrations of vapor, so that the flame reassumes its classical Burke-Schumann shape.

It is also instructive to the understanding of these two-phase diffusion flames to examine the behaviour of the temperature contours as a function of time. In Figure 6 this behaviour is illustrated. In keeping with the description of the role of the spray it is observed that for

$\tau \geq 0.33$ in the post-flame region the temperature contours T^* corresponding to 0.20 and 0.22 encompass a much greater area than in the first two time frames, $\tau = 0.175, 0.25$. This is indicative of heterogeneous droplet combustion beyond the homogeneous flame front. As τ reaches 0.62 and 0.76, the area can be seen to be diminishing in size as a state of negligible or no post-flame burning is approached.

10. Conclusions

We have developed a preliminary model of a fluctuating spray-diffusion flame. Numerical calculations which are based on the analytic solution indicate that the temporal flame envelope behavior is sensitive to the vaporization Damkohler number defined herein (and, therefore, to liquid-fuel volatility and/or droplet size). In all instances, some form of flame collapse and resurrection was observed. This type of behaviour is qualitatively in agreement with experimental observations. The intimate role of the spray of droplets in the dynamics of the pulsating flames has been elucidated. However, the next issue that must be addressed is that of the fluid-mechanical and/or heat- and mass-transfer mechanism that, in practice, produces such flame-envelope fluctuations. This matter is currently under investigation.

Acknowledgements

J.B. Greenberg wishes to thank the Fund for the Promotion of Research at the Technion for providing partial support for this research, and Michal Greenberg for help with the Figures. The dedicated technical assistance of I. Rappaport is also gratefully acknowledged. Further partial support was provided by grant No. 94-00012/1 from the United States-Israel Bi-national Science Foundation, Jerusalem, Israel.

Notation

c	L/R
C_p	specific heat
$\bar{C}_p$	sectional vaporization coefficient
$\mathcal{C}_o, \mathcal{C}_F$	fuel vapor free and oxygen free regions, respectively
D	diffusion coefficient
E	coefficient of evaporation
f	function describing time-dependent supply of liquid fuel
$\bar{f}$	constant (Equation (15))
$\mathcal{H}$	Heaviside function
L	radius of inner duct
m_d	mass fraction of fuel droplets
m_i	mass fraction of species i
m	defined in (4)
m_{totfuel}	total initial fuel mass fraction
n	integer
Q	heat of reaction
R	radius of outer duct
S_i	source term for species i
t	time
T	temperature
T^*	normalized temperature
T_o	reference temperature
u	axial velocity component
v_L, v_H	volumes of smallest and largest droplets in spray
V	Normalized initial mass fraction of oxygen
x, y	coordinates

Greek symbols

Δ	Damkohler number for vaporization
γ, γ_d	normalized mass fractions for gases and droplets
γ_T	Schwab-Zeldovich variable, Equation (12)
ν	stoichiometric coefficient
ξ, η	nondimensional coordinates
τ	nondimensional time
Ω	nondimensional frequency
λ	thermal conductivity

References

1. J. Ortman and A.H. Lefebvre, Fuel distributions from pressure-swirl atomizers. *AIAA Journal of Propulsion and Power* 1 (1985) 11–15.
2. T.J. Rosjford and S. Russell, Influence of fuel spray circumferential uniformity. AIAA Paper AIAA-87-2135, AIAA/ASME/ASEE 23rd Joint Propulsion Conference, San Diego, California (1987).
3. T.J. Rosjford and W.A. Eckerle, Alterations of fuel spray distribution by nozzle airflow. AIAA Paper AIAA-88-3140, AIAA/SAE/ASME/ASEE 24th Joint Propulsion Conference (1988).
4. J.B. Greenberg, Influence of initial nonuniform spray patterns on quasi-monodisperse droplet diffusion flames. ASME National Heat Transfer Conference, in R.K. Shah ()ed.) Vol. 106, *Heat Transfer Phenomenon in Radiation, Combustion and Fires* (1989) pp. 543–550.
5. N.A. Chigier, C.P. Mao and V. Oechsle, Structure of air-assist atomizer spray. Paper 7-6A, CSS/WSS/Combustion Institute Spring Meeting (1985).
6. J. Bellan and K. Harstad, Turbulence effects during evaporation of drops in clusters. *International Journal of Heat and Mass Transfer* 31 (1988) 1655–1668.
7. C. Presser, A.K. Gupta, H.G. Semerjian and R.J. Santoro, Droplets/air interaction in a swirl-stabilized spray flame. 2nd ASME/JSME Thermal Engineering Conference 1 (1987) 73–83.
8. C.P. Mao, G. Wang and N.A. Chigier, An experimental study of air-assist atomizer spray flames. 21st Symposium (International) on Combustion, The Combustion Institute, (1986) 665–673.
9. N. Dombrowski, W. Horne and A. Williams, The formation and combustion of iso-octane sprays in hot gases. *Combustion Science and Technology* 9 (1974) 247–251.
10. Y. Levy and D. Bulzan, On the oscillations of combustion of a laminar spray. *Combustion and Flame* 100 (1995) 543–549.
11. I. Kimura, Stability of laminar-jet flames. Tenth Symposium (International) on Combustion, The Combustion Institute (1965) 1295–1300.
12. J.D. Buckmaster and N. Peters, The infinite candle and its stability – a paradigm for flickering diffusion flames. Twenty-First Symposium (International) on Combustion (1986) 1829–1836.
13. T.W. Park, S.K. Aggarwal and V.R. Katta, Effect of gravity on the structure of an unsteady spray diffusion flame. *Combustion and Flame* 99 (1994) 767–774.
14. S.P. Burke and T.E.W. Schumann, Diffusion flames. *Industrial and Engineering Chemistry* 20 (1928) 998–1004.
15. A.J. Yule, C.Ah. Seng, P.G. Felton, A. Ungut and N.A. Chigier, A study of vaporizing fuel sprays by laser techniques. *Combustion and Flame* 44 (19892) 71–84.
16. Y. Tambour, A Lagrangian sectional approach for simulating droplet size distribution of vaporizing fuel sprays in a turbulent jet. *Combustion and Flame* 60 (1985) 15–28.
17. J.B. Greenberg, I. Silverman and Y. Tambour, On the origins of spray sectional conservation equations. *Combustion and Flame* 93 (1993) 90–96.
18. J.B. Greenberg, D. Albagli and Y. Tambour, An opposed jet quasi-monodisperse spray diffusion flame. *Combustion Science and Technology* 50 (1986) 255–270.
19. C.E. Polymeropoulos, Steady state vaporization and ignition of liquid spheres. *Combustion Science and Technology* 8 (1973) 111–120.
20. C.K. Law and W.A. Sirignano, Unsteady droplet combustion with droplet heating – II: conduction limit. *Combustion and Flame* 28 (1977) 175–186.
21. C.K. Law, Adiabatic spray vaporization with droplet temperature transient. *Combustion Science and Technology* 15 (1977) 65–74.
22. M. Labowsky, Calculation of the burning rates of interacting fuel droplets. *Combustion Science and Technology* 22 (1980) 217–226.
23. N.A. Chigier, *Energy, Combustion and Environment*. McGraw-Hill (1981) 496 pp.
24. J.B. Greenberg and Y. Tambour, Far-field coalescence effects in polydisperse spray jet diffusion flames. 21st Symposium (International) on Combustion, The Combustion Institute (1986) 655–663.
25. Y. Tambour, I. Silverman and J.B. Greenberg, Effect of initial multisize droplet distributions on polydisperse spray jet far-field diffusion flames. *Atomization and Sprays* 4 (1994) 565–582.
26. J.B. Greenberg, The Burke-Schumann flame revisited – With fuel spray injection. *Combustion and Flame* 77 (1989) 229–240.
27. J.B. Greenberg, Spray diffusion flames with arbitrary initial droplet velocity distributions. *Combustion Science and Technology* 75 (1991) 13–30.
28. G. Chen and A. Gomez, Counterflow diffusion flames of quasi-monodisperse electrostatic sprays. 24th Symposium (International) on Combustion, The Combustion Institute (1992) 1531–1539.
29. J.B. Greenberg and I. Shpilberg, Lewis number and vaporization effects in spray diffusion flames. *Atomization and Sprays* 3 (1993) 437–462.